The pendulum

School of Physics

Introduction

In this experiment, you will use an observe the oscilliatory nature of a pendulum and the factors on which its period depends. You will then determine a value for gravitational field strength g, and look at the accuracy of your measurement in terms of uncertainties.

Assumed Knowledge

PH12-12/Advanced Mechanics

Students:

- apply qualitatively and quantitatively Newton's Law of Universal Gravitation to:
    - determine the force of gravity between two objects $F = \frac{GMm}{r^2}$
    - investigate the factors that affect the gravitational field strength $g = \frac{GM}{r^2}$
    - predict the gravitational field strength at any point in a gravitational field, including at the surface of a planet (ACSPH094, ACSPH095, ACSPH097)

What to do before you get to the lab

1. Read up on the 'Assumed Knowledge'
2. Read the 'Preliminary Information' and complete any *P* questions.
3. Read the rest of the exercise so that you will know that to do in the laboratory.
4. Watch the lab video https://youtu.be/8PxXkxqVrWA
   (Or search the *UNSW Physics* channel on YouTube for *"Pendulum"*)

Preliminary information

Gravitational field strength, g

Gravity is an attractive non-contact force that emanates from and is exerted on objects with mass. Gravity is applied by means of a gravitational *field* that surrounds all objects with mass; when an object experiences gravity, it is often referred to as its *weight*.
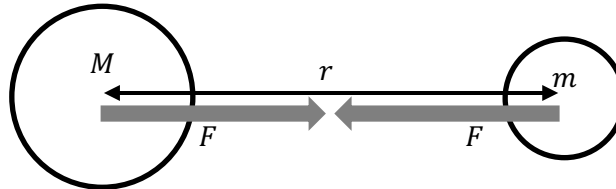
The strength of the force between two objects with masses M and m is given by

$$F = G\frac{Mm}{r^2}$$

where r is the distance between the centres of the objects and G is the universal gravitational constant, 6.67×10^{-11} N m^2 kg^{-2}.

The geometry of the gravitational field is such that its strength decreases in proportion to the inverse of the square of distance (i.e. obeys the inverse square law).
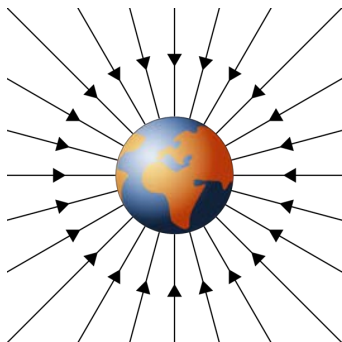

Figure 1

At large scales and distance, the gravitational field of the Earth is radial and its strength obeys the inverse square law.
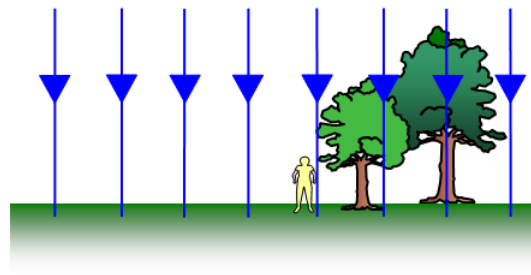

Figure 2

Close to the surface of the Earth and for small displacements from the Earth's surface, the Earth's gravitational field is approximately uniform.

On Earth, we feel the Earth pulling down on us, but we do not seem to feel the Earth being pulled up towards us. This is because the Earth is much more massive than we are. In the situation where a small object is nearby to a large dominant object, then we can simplify the situation to one where we just focus on the behaviour of the smaller object in the gravitational field of the larger one.

The weight of an object with mass m can be determined using the simplified equation $F = mg$

where g is the gravitational field strength. On Earth's surface, this value is commonly accepted as about 9.8 N kg^{-1}.

Question P1

Show that surface gravitational field strength can be given by the equation

$$g = \frac{GM}{r^2}$$

Question P2

Why might gravitational field strength vary between different locations on the Earth's surface?

The pendulum

A pendulum can be used to measure the local gravitational field strength. The pendulum we will use consists of a mass at one end of a string, with the other end fixed to a stand. The length of the pendulum can be varied.

It has been shown that the period of a pendulum T (the time to complete one back and forth swing) is related to its length l. The mass of the pendulum and the angle of the swing (for small angles) does not affect its period.

The equation for the pendulum's period is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where g is gravitational field strength.

In this experiment, we will be finding a value for g using linear regression (i.e. by using a line of best fit).

Question P3

In order to obtain a line of best fit, the pendulum equation needs to be *linearized*. Rearrange the equation into the form that resembles the general equation for a straight line.

Question P4
Which variables should be plotted along the horizontal and the vertical axes of the graph?

Horizontal axis:	Vertical axis:

Question P5
Write an expression that relates the gradient of this graph with gravitational field strength, g.

In the laboratory

Equipment

- Pendulum apparatus
- Boss head
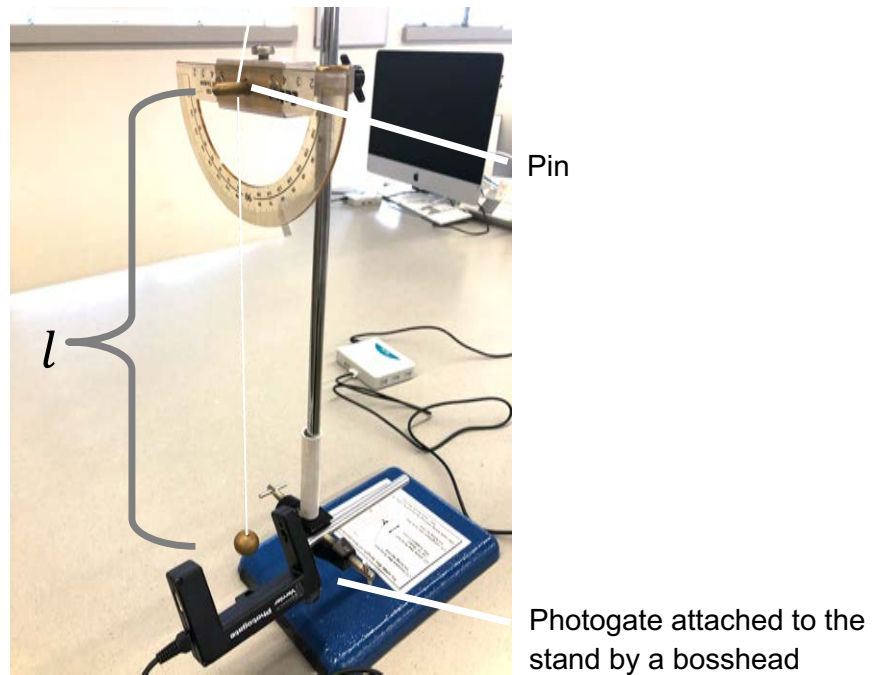- Photogate and LabQuest interface
- Metre rule

Figure 3

The pendulum apparatus

The pendulum apparatus consists of a brass ball on the end of the string. The string itself is of a fixed length – you will not need to cut the string to length. The length of the pendulum can be varied by raising or lowering the brass pin through which the string passes – just loosen the thumbscrew where the pin assembly attaches to the retort stand and slide it up or down. You can also control the initial swing angle by referring to the protractor in which the pin is set.

Method

Step 1 – Set up

Set up the photogate so that it rests on the bench with its jaw facing open-end up and the ball of the pendulum swings through it. Ensure that the ball breaks the infra-red beam as it swings through the bottom of its swing arc (see figure 3). Connect the photogate to the DIG1 port of the LabQuest interface.

Open *LoggerPro* on the computer. The program should have already recognised that a photogate has been connected. At the moment, it is in its default motion detecting mode. There is a special pendulum mode we can use to measure periods of a pendulum.

Changing photogate to 'pendulum mode'

Select ***Experiment > Set Up Sensors > LabQuest Mini: 1*** from the menu.
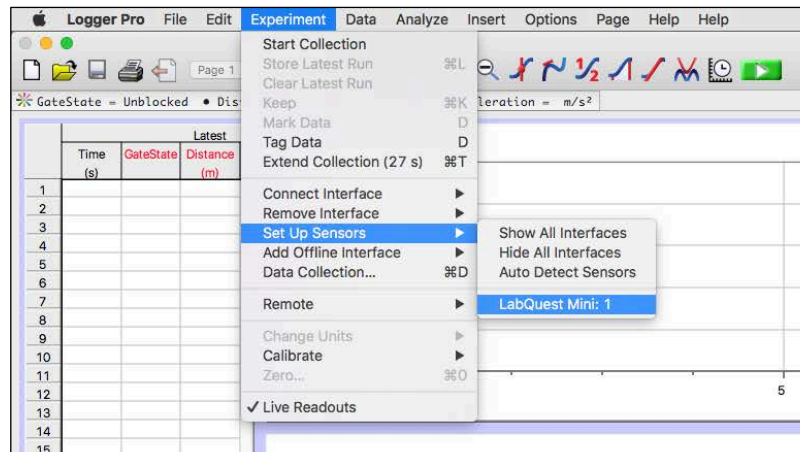
Click on ***DIG/SONIC* 1 (Photogate)** and select ***Pendulum Timing*** from the menu. Click the **×** to close the box.
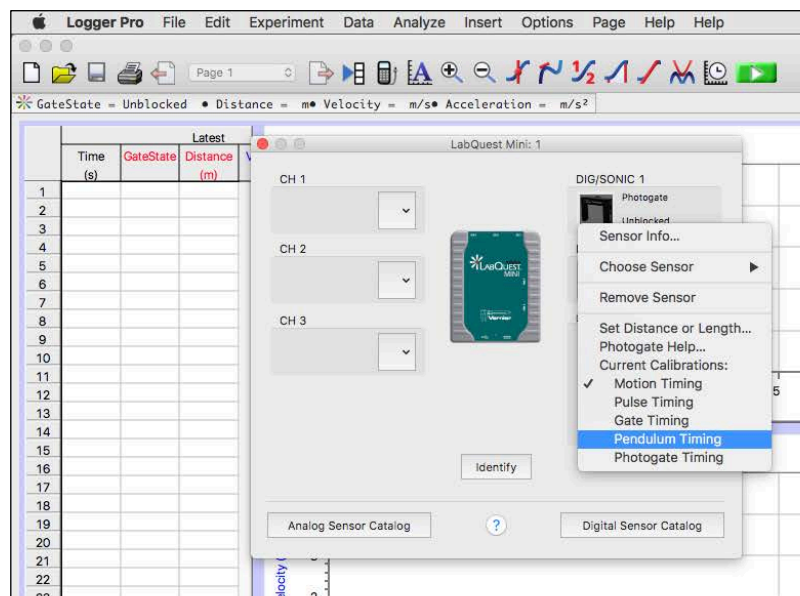
We are now ready to start collecting data.

Step 2

Adjust the pin assembly so that the length of the pendulum is 20 cm. Pull the ball to one side (about 10 degrees from vertical) and click the collect button ▶.

Release the ball and allow it to swing freely. *LoggerPro* will run for about 20 seconds, measuring the period of each swing, and then automatically stops.

Step 3

Once LoggerPro has finished collecting data for a run, you may see a line graph showing that each period was not necessarily the same. You may need to scale the axes in order to see this detail – click on the *"Autoscale"* button in the toolbar to do this.

We will need to find the average period for the pendulum for this length. Click and drag a rectangle around the line of the graph, leaving out the first and last data point as these may contain errors. Then click on the *"Statistics"* button.
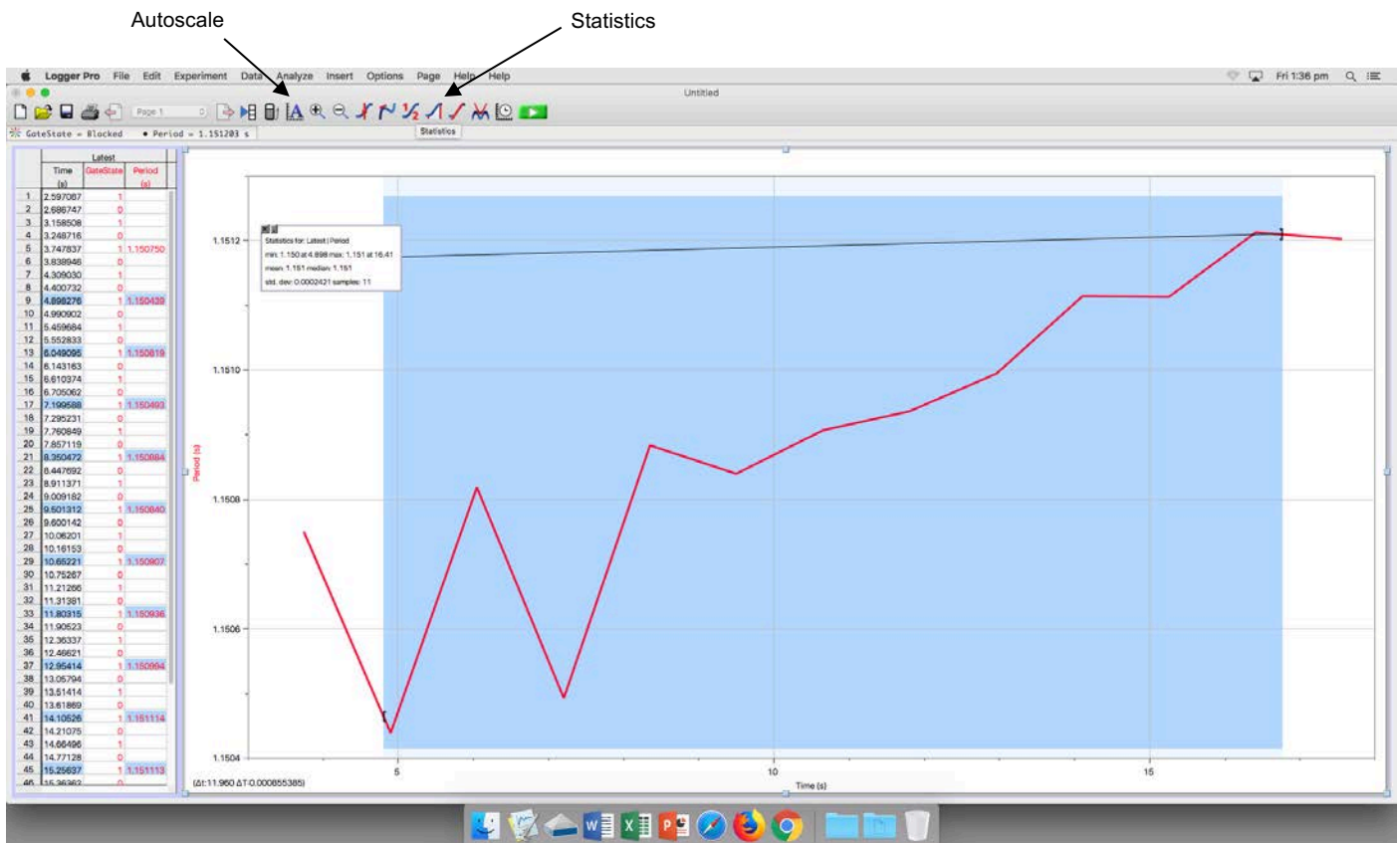
You should see a box pop up on the graph like the one shown above. Record the *mean* in the space on the next page.

Repeat the steps to measure the average period for this length two more times so you have a total of three trials for $l = 20$ cm.

Also, calculate the uncertainty in the period, ΔT.

	Period (s)				
Pendulum length	Trial 1	Trial 2	Trial 3	Average T	Uncertainty ΔT
0.20 m					

To calculate the uncertainty in the period,

$$\Delta T = \pm \frac{T_{max} - T_{min}}{2}$$

Step 4

Adjust the length of the pendulum so that $l = 25$ cm. As before, measure the period of the pendulum at this length three times. Record the data in the table in the *Results* section (including the first set of trials from the previous step).

Step 5

Collect and record three trials of period for pendulum lengths of 30 cm, 35 cm, 40 cm, 45 cm and 50 cm.

Results

Table 1.1

After you have entered your results, calculate period squared (T^2) for each length and its uncertainty ΔT^2.

Length (m)		Period (s)					Period squared (s²)	
l	Δl	Trial 1	Trial 2	Trial 3	Average	ΔT	T^2	ΔT^2
0.20	0.5							
0.25	0.5							
0.30	0.5							
0.35	0.5							
0.40	0.5							
0.45	0.5							
0.50	0.5							

Combining uncertainties

When you use uncertain numbers in calculations, the product is more uncertain. The way to calculate the uncertainty for $a = b \times c$ is

$$\frac{\Delta a}{a} = \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

UNSW
SYDNEY

Results 1.2

Rewrite this equation so that you can use it to calculate the uncertainty in period-squared, given that you have collected data for T and ΔT, and that you can easily calculate T^2.

Analysis

We will use Microsoft Excel to plot a line of best fit. Use the *Linear Plot with uncertainties* Excel file to plot a graph of *period squared vs length* (i.e. $T^2 vs\ l$).

To access this Excel file,

> *Desktop*
>> └ *First Year*
>>> └ *Drag to desktop and rename as Experiments*
>>>> └ *Fundamentals experiments*
>>>>> └ 🗒 Linear plot with uncertainties.xls

and then follow the instructions.

Write the equation for the line of best fit in the space below

Y = [   ]   X   +   [   ]

Write the equations for the two lines of worst fit in these spaces below

Y = [   ]   X   +   [   ]

Y = [   ]   X   +   [   ]

Analysis 1.3

Calculate the uncertainty in the gradient, where

$$\text{Uncertainty in gradient} = \pm \frac{\text{worst gradient}_{max} - \text{worst gradient}_{min}}{2}$$

Analysis 1.4

Use your equation from *Question P5* to calculate a value for g.

Analysis 1.5

Calculate the uncertainty in your value for g. The percentage uncertainty in the gradient is the same as the percentage uncertainty in g, i.e.

$$\frac{\Delta a}{a} = \frac{\Delta b}{b}$$

Discussion

Discussion 1.6

How close is your value to the accepted value for g? Has your experimental technique been precise enough to measure it accurately?

Discussion 1.7

What are some reasons why this experiment, as you have carried it out, may not measure the same accepted value for g?

UNSW
SYDNEY

Discussion 1.8
What are some reasons why the value for g itself might really not be the same as the accepted value?

Conclusion

Write a summary of the things that you have achieved in this activity.

END OF THIS ACTIVITY

UNSW
SYDNEY